

$$K = \int_0^u F dx = mc^2 - m_0 c^2$$

$$\begin{aligned} K &= \int_0^u F dx = \int_0^u \frac{d}{dt}(mv) dx = \int_0^u d(mv)v = \int_0^u (mdv + vdm) v \\ &= \int_0^u (mv dv + v^2 dm) \quad (1) \end{aligned}$$

$$\text{da: } m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{cioé: } m^2 c^2 - m^2 v^2 = m_0^2 c^2$$

differenziando:

$$2m dm c^2 - 2m dm v^2 - m^2 2v dv = 0$$

dividendo per 2m:

$$mvdv + v^2 dm = c^2 dm$$

e sostituendo nella (1)

$$\int_{m_0}^m c^2 dm = c^2(m - m_0)$$