CONTINUITY AND DISCONTINUITY. ON METHOD IN LEONARDO DA VINCI MECHANICS

1. General outline of the problem

The development of science shows its continuity–discontinuity and progressiveness. According to recent studies, in the last century the research about the foundations of science seems to have been forwarded increasingly by programs of research, more than through the implementation of a basic theory. Several competitive research programs have characterized the turn of 19th spanning throughout the 20th century. As a matter of fact, discovery seems recast in its scientific value whenever it has not undergone the filter of different approaches and scientific theories, even in conflict to each other, since their foundations; so the evaluation itself of the scientific value of a theory cannot be an absolute one. It is enough to remark that the interesting intellectual effort proposed by Thomas Kuhn (1922–1996) based on the idea of scanning scientific structures in the history of science which can establish themselves as paradigm or produce a replacement of an old framework. However, today we know that program was not completely adequate to

∗ I have to thank prof. Danilo Capecchi for the precious discussions and prof. Robert Zaborowski for last reading and suggestions.


understand continuity and discontinuity and/or commensurability in the (historical) development of science. In fact, he used a unique, Newtonian paradigm to analyze the development of theories: in this sense the theories having different foundations as well as Renaissance statics and modern mechanics, Newtonian mechanics and Lagrangean one, chemistry\(^1\) and thermodynamics were scarcely considered in his research\(^2\). Consequently, the Kuhnian program, which intended to explain all the scientific revolutions through the conceptual scheme of the Newtonian dynamics, found its path obstructed by the history of the evolution of the black body\(^3\). As a matter of fact, when Kuhn faced this matter to motivate the birth of quanta in *Black Body Theory and the Quantum Discontinuity* he had to give up\(^4\) the Newtonian paradigm that used to be his main inquiry category. The use of other kinds of categories by means of logic and mathematics let the eventual *revolutionary* or *normal* logical character come out in a scientific theory (organization) further than his formal conception of infinite in the use of mathematics. The latter gives us a hint about the choice of formalism and continuous or discontinuous scientific progress\(^5\).

2. Excursus on the scientific and cultural environment

The privileged geographical position of Italy in the Mediterranean caused interesting commercial exchanges with Africa and the Middle East that favoured the free circulation and the widespread of Greek works throughout Italy and Northern Europe\(^6\). On the other hand, when the Turks captured Constantinople (1453) many Greek scholars moved to Europe (several of them to Italy as well), taking with them important manuscripts and making the knowledge of the classical culture more accessible, compared with the past 12\(^{th}\) and 13\(^{th}\) centuries. The translation into Latin straight from the Greek language made their contents\(^7\).

---


\(^2\) A. Drago & P. Cerreta, *Il programma storiografico di Kuhn* ...


\(^7\) It is also necessary to remark that the same Greek writings were originated from as many primary sources in Persian and Arabian language. See G. Ferriello, *L’estrazione delle acque nascoste trattato tecnico–scientifico
Continuity and Discontinuity. On Method in Leonardo da Vinci’s Mechanics

More reliable. Reliability increased thanks to the invention of movable type printing (ca. 1450) by Johann Gutenberg (1400?–1467?). Approximately, since 1474 they started to print works of mathematics, astronomy and astrology in Italy; the edition by Giovanni Campano in 13th century might have been one of the first translations of the Elements by Euclid (1482) in its Latin version. In such a climate and until Renaissance the image of the new scientist, seen also as a student of natural phenomena, emerged. He was seen as a new type of scientist, re–born and re–qualified, not just an interested and clever astrologer and medieval theologian. Above all he looked now independent from a hypothetical and general pre–established design. However, the reconcilement between the divine plan and the new mathematical truths could converge into an outlined project, still divine under many aspects, considering God as the engineer who had planned a cosmological design in mathematical and geometrical terms. Koyré in his Newtonian studies wrote:

Once more the book of nature seemed to reveal God, an engineering God this time, who not only had made the world clock, but who continuously had to supervise and tend it in order to mend its mechanism when needed (a rather bad clockmaker, this Newtonian God, objected Leibniz), thus manifesting his active presence and interest in his creation.

God as an engineer allowed a certain chance of studying the divine product that is nature interpreted in mathematical terms, since in this way the object of study was still confined to a religious matter. In fact, this would explain why, among other things, the majority of the Renaissance scientists were theologians as well, who preferred to inquire into nature instead of the Holy Scriptures. Therefore each discovery or mathematical invention was seen as the product of God’s engineering work. Though this new way of conceiving it science was limited to the learned and the rich only, since they

---


5 A. Koyré, Newtonian studies, p. 21.

6 See E. Grant, La Scienza nel Medioevo, pp. 25–54.
had a knowledge of Latin and Greek. The spread of the new culture by print was hampered by two factors. First, a lot of technicians, such as architects and engineers, would have probably welcomed the application of geometry and mathematics as theoretical science to arts, navigation and architecture but the precarious diffusion of school education did not give the pioneers of scienza activa access to the necessary scientific heritage. In fact:

The book De Architectura by the Roman architect and engineer Vitruvius (first century AD) became known in the 12th century; nearly all the works of Archimedes (3rd century BC) were translated by William Moebere in 1269, from Greek into Latin [...]. But these writings had no influence on the practice of medieval craftsmen who did not understand Latin1.

Therefore, according to some thought currents of history of mathematics2, the expectation about the spread of the classical culture, instead of encouraging the highest erudition among mathematicians and, in general, of scientific topics3, paradoxically seemed to exclude just the new-born class of scientists—mechanics who, far more numerous than theoretical scientists, felt a strong interest in the introduction of mechanical devices4 or of calculating ones within their treatises. Second, the other factor is a more philosophical one. By then theoretical knowledge was the only one to be considered full and definitive, therefore experience was meant to be of secondary use, so the discoveries of technicians were ignored, eventually causing a strange regression toward the medieval culture typical of the Scholastics of 12th century. In particular, due to the lack of mathematical devices, technicians would feed their knowledge through the development of so-called procedures by comparison. Models by similitude were typical, after daily practice and based upon make mistakes and correct5, almost to represent a sort of a practical handbook of architecture. Sometimes, as regards some authors, we can guess interesting, though embryonic, references on the study of the strength of materials related (geometrically) to the structure, the air quantity or to the height of the columns6. The scientific applications will flow into the new technology and will require more and more the integration of local activities and the managing skill of the artisans7. This integration and the new


3 For economy of space I will not deal with the meaning of science in this historical time. As a matter of fact, the Aristotelian paradigm marked a certain distinction among what science was and what it was not.

4 On the epistemological role played by scientific instruments one can see a recent book: S. D’Agostino, Gli strumenti scientifici e la scienza, Barbieri Editore, Manduria 2005.


6 See V. Marchis, Storia delle macchine, pp. 46–47.

7 Charles Singer (1876–1960), Trevor Williams and Thomas Derry, tried to study the role played by
reference to the Euclidean geometry will bring together with other physical–mathematical factors – that will be the casestudy of the present thesis – to the realization of the first projects, after the aestimatio model, that is approximated and designed on the spot\(^1\).

Among non–humanists approaching mathematic studies and pure mechanics, Niccolò Fontana, called Tartaglia (1500?–1577)\(^2\) is the most outstanding figure. He could be defined as the scholar who by then best represented the crucial turn from the learned mathematician to the practical one, also endowed with skills in the field of mechanics and architecture. He was able to pick up with great attention from the architects’ and engineers’ practical knowledge, especially the military ones, theoretical matters which later were to prove crucial for the projections that were then real elaborated by others, even if some never mentioned Tartaglia\(^3\), e.g. see the case of Galilei’s work, ‘Trattato di Fortificazione’\(^4\) and in his theory of projectiles.

3. Leonardo da Vinci’ cultural background: continuity or discontinuity?

The figure of Leonardo da Vinci\(^5\) is set in the historical period mentioned...
above. Generally, when the so–called *scientia pratica* of the Renaissance is referred to, we are reminded of engineers and, consequently, of Leonardo da Vinci (1452–1519), the great scholar who sums up a multiplicity of competences that nowadays would be considered as different crafts: from the engineer, architect, scientist to the artist. Although some studies, such as from Pierre Duham (1861–1916), Roberto Marcolongo, (1862–1943), Clifford Truesdell (1919–2000) and Bertrand Gille (1920–1980) suggest a review of Leonardo da Vinci’s role as a genius, in favour of a more human figure of a *learned man*, endowed with a quick intelligence, e.g. not all his designs about machines sprang out straight of his fantasy. Thus:

*Leonardo da Vinci is perhaps overrated for his contributions to science, since his was more the mentality of the engineer; his notebooks are neither systematic nor lucid expositions of physical concepts. Yet he too supplied an important ingredient, wrestling as he did with practical problems of mechanics with great genius and technical ability. He brought alive again the tradition of Jordanus Nemorarius and Albert of Saxony, and his speculations on kinematics and dynamics, if inconclusive, reveal how difficult and elusive were the conceptual foundations of mechanics for its early practitioners*.  

Further, that is also confirmed by the discovery of a manuscript by the great architect and engineer Francesco di Giorgio Martini from Siena (1439–1501) where in the part on machines several notes in Leonardo da Vinci’s hand were re–discovered. Taking into account that modern historiography reached the conviction that Leonardo got his results in part from other sources or that he would have written them previously together with other authors, we can

---


2 To support these hypotheses in the 1970’s Gille mentioned a *Manuscript* by architect–engineer Francesco di Giorgio Martini from Siena. It is *Ashburhamiano N. 361* and placed in Laurenziana Bibliotheca of Firenze. The manuscript was not complete but included most part of mechanics and notes by Leonardo da Vinci. This aspect brings Gille to conclude that for a long time everyone thought that the author of the manuscript was just him, Leonardo da Vinci. See B. Gille, *Leonardo e gli ingegneri del Rinascimento*, transl. A. Carugo, Feltrinelli, Milano 1972, p. 128. [original edition: B. Gille, *Les ingénieurs de la Renaissance*, Hermann, Paris 1964. On this theme see also L. Russo, *La rivoluzione dimenticata. Il pensiero scientifico greco e la scienza moderna*,...
reasonably make the hypothesis that the abundance of materials about his
scripts and the lack of it in other cases could also be due to greater care when
searching the documents of the brilliant scholar. So it is difficult to make a
hypotheses about an artist’s inspiration. In fact, without a proper method of
historical inquiry it is not so easy to deduce from his manuscripts what one
author takes from another and what really represents scientific continuity or
discontinuity:

After Aristotle and Ptolemy the idea that the earth
moves – that strange, ancient, and “entirely ridicul-
ous”[1] Pythagorean view – was thrown on the
rubbish heap of history, only to be revived by Copernicus and to be forged by him into a weapon for the
defeat of its defeaters. The Hermetic writings played
an important part in this revival, which is still not
sufficiently understood[2], and they were studied with
care by the great Newton himself[3] such develop-
ments are not surprising. No idea is ever examined in
all its ramifications and no view is ever given all the
chances it deserves. Theories are abandoned and
superseded by more fashionable accounts long before
they have an opportunity to show their virtues.[4]

In other words but in the same vein, Mach wrote:

1. That branch of physics which is at once the oldest
and simplest and which is therefore treated as
introductory to other departments of this science, is
concerned with the motion and equilibrium of masses.
It bears the name mechanics. 2. The history of the
development of mechanics, is quite indispensable to a
full comprehension of the science in its present
condition. It also affords a simple and instructive
example of the process by which natural science
generally is developed.[5] [...]. They that know the
entire course of the development of science, will, as a
matter of course, judge more freely and more
correctly of the significance of any present scientific
movement than they, who limited in their views to the
age in which their own lives have been spent, contem-


1 Ptolemy quoted by Feyerabend: P. Feyerabend, Against Method, Humanities Press, London 1996, p. 35,
n. 4.

2 Ptolemy quoted by Feyerabend: P. Feyerabend, Against Method, p. 35, note n. 5.


4 P. Feyerabend, Against Method, p. 35.

plate merely the momentary trend that the course of intellectual events takes at the present moment.\textsuperscript{1}

Leonardo is an artist but also a technician and a scholar and [...] it would be a mistake, assuming a position systematically too antithetic to the official thesis, to assimilate his notes to a definitive work of art\textsuperscript{2}. Then, we must say that an indirect continuity in a bend toward science shown by Leonardo emerges when considering that the themes he dealt with had already been studied in early 1400 by Mariano Taccola from Siena who was interested in the scripts of mechanics and military technology of Pneumatica\textsuperscript{3} by Philon of Byzantium (280–220 B.C.). As the majority of engineers by that time, Leonardo also studied the engineering works by Heron from Alexandria (I c. B.C.) though considered useless toys. On the other hand, they got enthusiastic before the futuristic technical designs by Leonardo in that when not copying it, they were strongly influenced by them, such as Hero’s engine, windwheel, vending machine, force pump, Heron’s fountain, et al.\textsuperscript{4} Gille ends up his book with a hope:

\textit{All our engineers were men of war. Such statements of the obvious have the uncomfortable habit of often being true. Yet the sixteenth century had passed beyond warlike preoccupations and had constructed a complete technical system, just as it had built a new scientific system. More than their quest for deadly power, more than the amusements and the love of images, what has attracted us in these men is the difficult apprentice ship they served in a new world. Much remains to do before we understand the processes of their thought, before we appreciate their hesitations and grasp the nature of their ignorance and their failures. We must underline their gradual distortions of accepted truths, their difficult departures from the traditional paths, in order to give them credit for having [...] unique advance in the history of thought. [...] But the enquiry remains open: it might bring to light other works still languishing in the dust of libraries, it might also provide a more precise analysis of the notebooks which have never been published and which are full of information.\textsuperscript{5}}

Nowadays Leonardo da Vinci’s cultural matrix seems clear. Historians agree in considering the Aristotelian physics as the main source of his mechanics.

\textsuperscript{1} E. Mach, \textit{The Science Of Mechanics} ... , p. 7.
\textsuperscript{2} L. Russo, \textit{La rivoluzione dimenticata} ... , p. 282 [English transl. is mine – R. P.].
\textsuperscript{3} The Italian translation of \textit{Pneumatica} by Filon from Byzantium is reported in the first part of \textit{War and hydraulic machines in 15th century}. See L. Russo, \textit{La rivoluzione dimenticata} ... .
\textsuperscript{4} See L. Russo, \textit{La rivoluzione dimenticata} ... , p. 389 [English transl. is mine – R. P.].

Actually, even if Leonardo da Vinci’s research works concern almost exclusively the fields he practiced as a technician, a need of a mathematical–geometrical abstraction and of rationalization seems to emerge; apparently neglected until then by technicians, there was an exigency to define technique through observation and the mathematical explanation of phenomena. Nonetheless it is worth remarking that a consequence of this early form of *discontinuity* is the fact that Leonardo da Vinci’s method surely did not spring out of nowhere. It is rooted in the scientific tradition of the Aristotelian school, further than in the Archimedean one. More specifically, many are the traces of Aristotle’ thought² to be found in Leonardo, starting with the concept that the knowledge of universal things (the furthest from our senses, in contrast with the singular things which are the closest to our sensible perception) is acquired by means of reasoning based on primitive truths that cannot be proved; the latter can be known by induction, that is by means of data of the sensible perception stored in our memory:

*From sense, therefore, as we say, memory is produced, but from repeated remembrance of the same thing, we get experience, for many remembrances in number constitute one experience. From experience, however, or from every universal being at rest in the soul that one besides the many, which in all of them is one and the same, the principle of art and science arises, if indeed it is conversant with generation, of art, but if with being, of science.*³


At the same time, Leonardo draws on Archimedes’ *scientia*, in particular he shares the methodology based on the study of the equilibrium of bodies, that is he follows the rational criteria that the mathematician from Syracuse had set to determine the centres of gravity\(^2\). Leonardo introduces the concept of *pratika* as the basis of any of his studies, declining it either as observation, a study of buildings, of human anatomy and natural phenomena, or as an experiment aimed at checking up the calculations derived from his observation. On the other hand, he defines himself *discepolo della sperienza*\(^3\). To him, from experience we can derive, beyond good building practices, also rules that are not only the expression of aesthetic research but principally requirements for the proper performance of the *building organism*, considered at the same time as a living organism or a *macchina–ingegeo*\(^4\). With Leonardo, it very often recurs, perhaps for the first time, the idea of an absolutely efficient building–machine. Within it daily activities are made rational and mechanic: e.g. a fireplace automatically operated, a laundry, the model of a stable\(^5\). The building is conceived as a *living organism* but, at the same time, in a sense, taking Vitruvius’ concepts to the extreme, he suggests also the way round. In other words, living organisms too – men and animals – are turned into *macchine*. In this sense, he detects in any organism, living or not, a unity of process and function based on movement and considers animals as a human body and buildings as a whole of mechanical devices, that he calls *elementi macchinali*\(^6\). Bird is a device performing after a mathematical law\(^7\) [...] and nature cannot make animals move without ‘mechanical devices’.\(^8\) Leonardo da Vinci’s considerations around such mechanical elements and his studies of anatomy are really interesting, proving study and performance methods very similar:

---

\(^1\) See L. da Vinci, *I libri di meccanica nella ricostruzione ordinata* p. 70.


\(^3\) C. Pedretti, *Leonardo. Le macchine*, p. 36.

\(^4\) In the Renaissance, as in the Old Age, the term *ingegeo* pointed out the mechanism of a machine, and by extension, the machine itself. For Vitruvius, e.g. the *ingegeo* is a machine that requires only one person to be put at work. The term *building* could apply to the concept of machine (e.g for war or water) and vice versa to machine (e.g. Milano Dom machine). See C. Pedretti, *Leonardo architetto*, p. 309.


\(^8\) P. Galluzzi, *Gli Ingegneri del Rinascimento*, p. 192: *E la natura non può dar moto alli animali senza strumenti macchinali*. 

---
All such instruments will generally be presented without their armatures or other structures that might hinder the view of those who will study them.\textsuperscript{1}

This uniformity of treatment emerges in his drawings as well, either anatomic, where bones and muscles are handled as geometrical schemes of ingegni\textsuperscript{2}, or of machines and tools, in which relevant specific elements insist, such as the cannons–columns\textsuperscript{3} that seem to claim the universality of the planning project.

4. What kind of tradition until Leonardo?

Generally, the physical Aristotelianism and its mechanization\textsuperscript{4} can essentially be viewed as the whole of the knowledge based upon experience, concerning all outer phenomena of the sensible universe. Leonardo da Vinci’s mechanics, after the tradition of the Aristotelian physics, focuses on the motions that can occur in a mechanism as its study object. Under the aspect of logic and organization of the theory, Leonardo studies problems of engineering concerning his profession, e.g. strength of masonry in buildings, arch–thrust, wall cracking. From his studies it is evident\textsuperscript{5} that his reasoning, at first intuitive, later proceeds by analogy and approximation, by means of a series of experiences, in search of absolute rigour. In order to do this, approximation useful also in practical applications is sufficient. As a matter of fact, it might have been the early use of such approaches to kind of works previously approached by traditional means and hardly rational evaluation methods\textsuperscript{6}. Leonardo also studies the typical problems of mechanics that are nowadays solved by the building science, axial and flexional analysis of a wood beam: materials submit to strength by a column axially and flexionally loaded and strength of horizontal beams. Through a rough calculation and after experience, he is able to set the laws upon deformability of a wood beam, strength of beams with square or circular section, simply supported or clamped at one or two extremities, strength of composite beams. Leon Battista Alberti (1404–1472) and Francesco di Giorgio Martini also had dealt with the same problem, but Leonardo was among the first who tried to elaborate it by mathematical and geometrical approach. Leonardo aims to establish the universal laws ruling the attitude of the materials, of the building elements, of ingegni, in an attempt at reconciling what Paolo Galluzzi defines as two

\begin{itemize}
\item \textsuperscript{1} L. da Vinci, Codice di Madrid I, c. 82r.: Etali strumenti si figureranno in gran parte sanza le loro armadure o altra cosa che avessi a impedire l’occhio di quello che le studia.
\item \textsuperscript{2} The design by Leonardo upon the wings of the birds as part of machines hypothesized to fly. Codice Windsor RL 12656, ca. 1513–1514, Codice sul volo degli uccelli, f. 17r, ca. 1505.
\item \textsuperscript{3} The picture in the text: Macchine come elementi architettonici: studi di artiglieria in: L. da Vinci, Codice Atlantico, f. 28v.–a, ca. 1495–1497.
\item \textsuperscript{5} See B. Gille, Leonardo e gli ingegneri del Rinascimento, pp. 212–213.
\item \textsuperscript{6} See B. Gille, Leonardo e gli ingegneri del Rinascimento, p. 213.
\end{itemize}
mechanical traditions\textsuperscript{1}. First, mechanics, until the Middle Age, had generally neglected the practical application. Second, mechanics practiced and transmitted within the renaissance botteghe. The historical contextualization of tradition suggested by Gille\textsuperscript{2} is interesting, either concerning his interpretation on setting the authors until the Renaissance or thereafter by different traditions of mechanics, engineering and theoretical. I adapted and re-interpreted \textit{ad hoc} it in the following scheme, basing it on a distinction between Aristotelian and Archimedean modes\textsuperscript{4}, as thought currents until Leonardo, too.

<table>
<thead>
<tr>
<th>Theoretical and Archimedean tradition:</th>
<th>Engineering and Aristotelian tradition:</th>
</tr>
</thead>
<tbody>
<tr>
<td>J. Fontana (1393–1455?)</td>
<td>K. Kyeser (1366–?)</td>
</tr>
<tr>
<td>L. B. Alberti (1404–1472)</td>
<td>Brunelleschi (1377–1446)</td>
</tr>
<tr>
<td>F. di Giorgio Martini (1439–1502)</td>
<td>Mariano J. detto il Taccola (1382–1458?)</td>
</tr>
<tr>
<td>N. Tartaglia (1500?–1557)</td>
<td>Valturio (1413–?)</td>
</tr>
</tbody>
</table>

Leonardo da Vinci (1452–1519)

Starting from the \textit{method of comparison}, Leonardo compares the strength of beams and columns composed by same composition, different height and the same line section and vice versa. A beam is more rigid than another one if, under the same strength, it deforms less, or, in order to get an equal deformation, the introduction of a greater strength is required. In this case, the concept of strength is presented as stress (imagined–)concentrated in a point. This way it is possible to neglect what today we define such as a modulus of elasticity of bodies and, for the beams of the same section line, the momentum of inertia belongs to the section. By associating mathematical calculations and experiences, Leonardo da Vinci obtained that (1) in the case of supports with square section uniformly loaded at the top the strength to axial force is proportional to the surface of the section and inversely proportional to the ratio between height and the side of the square (or the radius of circular cross section)\textsuperscript{5}, (2) for a square section beams simply supported and loaded in the middle (\textit{mezzeria}) by a weight, the strength is inversely proportional to the weight, inversely proportional to the length and directly proportional to the square of the side of the section.

\textsuperscript{1} P. Galluzzi, \textit{Gli Ingegneri del Rinascimento}, p. 78. It is necessary to remark that upon this subject Drake already envisaged a distinction between two Italian schools of mechanics, based on the different geographical–cultural position and on the different affairs that every component of a school usually had in comparison to the others. See S. Drake & I. E. Drabkin, \textit{Mechanics in Sixteenth–Century Italy: Selections from Tartaglia, Benedetti, Guido Ubaldo, and Galileo}, University of Wisconsin Press, Madison 1996, pp. 13–16.

\textsuperscript{2} See B. Gille, \textit{Leonardo e gli ingegneri del Rinascimento}.


\textsuperscript{4} R. Pisano, \textit{Brief historical notes on the theory of centres of gravity}, pp. 934–941.

As regards his studies of strength of buildings, Leonardo is interested in the causes of sliding, collapse and crack:

When the crevice in the wall is wider at the top than at the bottom, it is manifest sign, that the cause of the fissure in the wall is remote from the perpendicular line through the crevice.\(^1\)

Furthermore, he spots the relation between the solidity of walls and the composition of the ground:

Parallel fissures constantly occur in buildings which are erected on a hill side, when the hill is composed of stratified rocks with an oblique stratification, because water and other moisture often penetrates these oblique seams carrying in greasy and slippery soil, and as the strata are not continuous down to the bottom of the valley, the rocks slide in the direction of the slope, and the motion does not cease till they have reached the bottom of the valley, carrying with them, as trough in a boat, that portion of the building which is separated by them from the rest.\(^2\)

Later, he adds suggestions and precautions to be taken for construction, e.g. raising the walls by successive layers using a mortar perfectly uniform for

---


\(^2\) Transl. J.–P. Richter in: *The notebooks of Leonardo da Vinci*, p. 770. L. da Vinci, Codice Arundel, r. 15: Li fessi paralleli sono universalmente generati in quelli edifici che si edificano i’ loci montuosi, li quali sien composti di pietre faldate con obliquo faldamento. E perché in tale obliquità spesso penetra acqua e altra umidità portatrice di certa terra untuosa e sdrucciolare, e perché tali falde non sono continue insino al fondo delle valli, tali pietre si mosvan portando con seco quella parte de lo edifizio che per loro si separa dal suo detto rimanente..
the regularity and the compactness of the whole. Leonardo da Vinci’s observations on beams concern either the axial and flexional behaviour. For this last issue he focused more attention on its buckling. These considerations are interesting though not always formal and precise experimentally. More in detail for the decline Leonardo is more concerned with deformability than strength. The reason could be that he refers mainly to the timber used in building war machines and ships. These beams are very thick and resistant to failure, so they are essentially dimensioned for deformation. In the following comments by Leonardo where he dealt with clamped and truss beam:

\[(P_I) \text{ One beam of 6 braccia is stiffer the double in its middle, than four equal sized beams of 12 braccia joined together.}\]

Based on recent research, the previous observation of Leonardo is in accordance with modern theory of elasticity of beams: a supported beam of constant section, highlighted by means of a concentrated force applied to mezzeria. The arrow $v$ is mathematically interpreted by the following formula:

\[v = \frac{1}{48} \frac{fl^3}{EI} (1),\]

where $E$ is the longitudinal modulus and $I$ the moment of inertia of the section. From the previous track and considering (1), from 6 to 12 arms, that is, doubling the light, the same section and force $f$ by formula above the arrow increased 8 times or rigidezza (rigidity) decreases 8 times. But 4 of 12 auctions arms absorb each $1/4$ of the force $f$ to which the arrow of four auctions together is equal to that of an individual charged with $1/4f$. The fall of each beam of 12 arms worth $1/4$ to 8 times so it is only 2 times that of an arm of 6. It is thus the result of Leonardo in $(P_I)$.

The $(P_I)$, however, does not appear as a general law, like: a beam composed of $n$ arms (braccia) is 8 times more rigid than a beam of $n/2$ arms. It seems rather the reporting of test results, idealized as reported to integer

---

2. They often were interested in the problem of force applied to the top of a beam supported at the side and clamped other one. See D. Capecchi, Storia della scienza delle costruzioni, Progedit Editore, Bari 2003.
4. The quotations are not clear and it looks like when they refer to a square section. See D. Capecchi, Storia della scienza delle costruzioni.
7. See D. Capecchi, Storia della scienza delle costruzioni.
numerical values. Moreover, the experimental capabilities of the times, and perhaps the approach to these studies did not allow to design and verify when the law is different from that expressed by integers. It should not be forgotten that the language of mathematics based on the proportions made it difficult to determine the structure for each physical law that applied sizes varying from each others. In the case of the cantilever Leonardo still makes a quantitative rule:

Let the support nb to be as much stiff in n as the support cd in c. The reason is that, since the support cd has a diameter double than the above quoted support ab, it results, thanks to fifth proposition, 4 times greater than support ab; but by considering equal distance from their fixed parts, it carries 4 times the weight. [...] Now one half [in length] of the support ab, that is nb, is of equal stiffness in n than in c, the extreme point of the support cd, because, if the dimension of one is 20 [or: 2?] times the dimension of the other, the other one is in the same proportion, then if subjected to equal weight, they will have equal resistance.¹

If the following sentence sia di tanta resistenza (a lot of strength), in the first part of the passage, and sia di pari sostentamento (equal sustenance) in the second part could by interpreted by have the same rigid situation and making the hypothesis that the reference to the less rigid 20 (snellezza) has only a limited value, then the P₂ provides the following law:

\[ v \propto f \frac{l^2}{H^2}, \quad (2) \]

Following analyses by Danilo Capecchi², I agree that expressing an opinion on the historical foundations of inconsistency from (P₁) to (P₂) is still complicated. One can assume that Leonardo considered that cantilevers could be subject to different laws than inflexional beams³. Or, more simply, that the report (2) could refer to two separate studies in time and of Leonardo da Vinci’s thought. In another proposition Leonardo seems to contradict Leonardo da Vinci’s rule (P₂) because he presents a case where the bodies vary according to an inverse proportion to l and not to \( l^2 \):

¹ See D. Capecchi, Storia della scienza delle costruzioni, p. 235. See also L. da Vinci, Codice Atlantico f. 86 verso a, new edition page: 234 v.: (P₂) Il sostentaculo n b sia di tanta resistenza in n, quanto il sostentaculo cd si sia in c. La ragion si è, che ‘l sostentaculo cd, per l’essere di duplicato diamitro nella sua grossezza al sostentaculo di sopra, ab, viene, per la quinta proposizione, a essere 4 tanti più grosso che esso sostentaculo ab; e però, in pari distanza dai loro, immobili fermentami, 4 tanti più peso sostiene. [...] Ora il mezzo del sostentaculo ab, cioè nb, sia in n di pari sostentamento a c. stremità del sostentaculo cd, perché, se ‘l diamitro della grossezza dell’uno entra nella sua medesima grossezza 20 volte, l’altro si trova in sè della medesima proporzione, e però a uno medesimo peso saranno di pari resistenza..

² See D. Capecchi, Storia della scienza delle costruzioni.

If you join together 9 beams having equal properties [sections and materials], you will find as much force and resistance, as in the ninth part [in length] of one of them. ab supports 27 and if they are 9 beams, so cd, that is the ninth part of them, can support 3; in this way, e f, that is the ninth part in length of cd will support 27 because is 9 time shorter than it.¹

The proposition is consistent and valid if it is assumed that – in this case – Leonardo, for the first time, takes into account the strength at break and not the deformability. Hence, on the basis of the laws of statics², the strength of the beams varies in inverse proportion to the length. In fact, the flexional momentum in case of power concentrated in the mezzeria varies linearly with the light of the beam and the momentum of strength is constant.

5. Concluding reflections

Aristotelian physics and the same approach to Archimedean mechanical method gives Leonardo da Vinci’s method a certain continuity with the ancient foundations of ancient science, but we should also remark that the contribution of Leonardo appears to offer a certain early vein of discontinuity with the tradition of the late Middle Ages as concerning the applications to architecture, especially from the methodological point of view. In fact:

Most historians are agreed that some break with Aristotle was necessary before the transition could be made from natural philosophy to science in the classical sense. [...] The fourteenth century marked the high point in optical experimentation and in the conceptual development of mechanics during the late Middle Ages.³

Indeed, compared to the other artisan–engineers who were his contemporaries, Leonardo had in fact the merit of trying a more thorough analysis of the causes and the general principles underlying the sensible phenomena. In short, Leonardo’s studies in mechanics can be summed up as follows:
a) search for a law based on the proportions and a thesis derived inductively from experience,
b) Leonardo does not submit a general law,
c) difficulty in conceiving the type of relationship between size and law expressed using integer numbers,
d) extension of the proposition to geometrically different beams. a method of

¹ L. da Vinci, Codice Atlantic A f. 152 recto b, new edition page: 410 r.: (P.) Tu troverai tal forza e resistenza nella collegazione di 9 travi di pari qualità, quanto nella nona parte d’una di quelle. a b sostiene 27 e son 9 travi, adunque e d, ch’è la nona parte d’essi, sostiene 3; essendo così, e l, che è la nona parte della lunghezza di d, sosterrà 27 perché è 9 volte più corto di lui. See also: L. da Vinci, I libri di meccanica nella ricostruzione ordinata ... , p. 236.

² It is independent in composition but assuming a flexional physical system and ignoring the case of cut–torsion.

³ W. A. Wallace, Experimental science and mechanics in the Middle Ages.
analogy and comparison conserving the ratios.

To summarize the details of the process of knowledge:

a) moves from qualitative (less deformability) to quantitative concepts (two beams have numerically equal rigidity),

b) replacement of new numerical values for new geometric configurations, conserving ratios,

c) he was not interested in absolute rigour, but used approximations as techniques usable in life,

d) by the technique of comparing (proportionality).

From the epistemological point of view Leonardo da Vinci’s contribution can be summarized as follows:

a) an attempt to unify the research on the sensible world, by applying the same criteria of representation and of a geometric–mathematical analysis to different fields: anatomy, architecture, industrial technique, painting,

b) centrality of sensory observation and experience, to be translated into design, which becomes an instrument of inquiry, explanation and graphic modelling,

c) a universal point of view; interpretation of the biological universe and of the mechanical one, considering systems governed by the same rules,

d) development of a process of theoretical and experimental research that starts from tasks and requirements of a practical nature and then develops theoretical considerations, compared with the classical and medieval primary sources of scientific knowledge, to be verified experimentally, in order to build up general mathematical rules applicable to specific cases,

e) pragmatic and realistic approach to the problems: Leonardo does not seek absolute rigour in the results of his research, but an approximation recognized as useful, clearly an attempt to rationalize all human activities, including his own.

Some classical sources

– Aristotle (384 a.C.–322 a.C.)
– Euclid (365 a.C.–300 a.C.)
– Archimedes (ca. 287 a.C.–212 a.C.)
– Thābit (826–901)
– Jordanus de Nemore (XIII sec.)
– Biagio da Parma (?-1416)
– ...

Direct observation and modelling of phenomena

Attempt to fix principles by comparison approach

Building and application by prototypes

Attempt provide mathematical formalization

Sources and methodology in Leonardo\(^1\)

\(^1\) It is a fact that by that time the translation of Archimedes by W. Moerbeke (1269) was not so largely
Leonardo seems to establish a closer relation between natural phenomena and theoretical science\(^1\), trying to elaborate also mathematical reasoning for the interpretation of the observed phenomena: *No human investigation can be called real science, if it cannot be mathematically demonstrated*\(^2\).

In conclusion, even in this brief essay, it is evident that the studies by Leonardo represent an important and partly correct attempt to formulate a general theoretical organization involving greater formalization – than his predecessors – which can clear up and preview, e.g., the deformability of bodies in mechanics and architecture. One of his aims was to avoid further planning mistakes to ensure the proper functioning of the *building–human organism* and of the *building–machine*. In this sense he is remote from his contemporaries. Later, toward the end of the Renaissance this new way to decide the theory that assumed a particular cultural value mainly proceeding towards an *analytical* perspective of conceiving mechanics that seemed to be coming to a crossroads: *physical or mathematical science*? That way another historiographic problem emerges: a crucial continuity–discontinuity problem appears when a theory is included in another theory, e.g., mathematics in mechanics (*rational mechanics*), astronomy in mechanics (*celestial mechanics*) mathematics in thermodynamics (analytical theory of heat), mechanics in engineering (*structural mechanics*).

---


\(^2\) Transl. C. Pedretti, *Leonardo. Le macchine*, p. 34: *Nessuna umana investigazione si può dimandare vera scienza, s'essa non passa per le matematiche dimostrazioni.*