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CONTINUITY AND DISCONTINUITY. AN EPISTEMOLOGICAL INQUIRY BASED ON THE USE OF CATEGORIES IN HISTORY OF SCIENCE[*]

1. Introduction

Generally, historical inquiry proceeds according to different levels of knowledge, both scientific and humanist, and through particular methods and instruments of research, for example in the history of foundations, in the scientific–cultural attitude of a particular historical period rather than in the historical–social consequences in the history of technologies. In the last century research about the foundations of science seems to have been forwarded increasingly by programs of research more than through the implementation of a basic theory. Several competitive research programs have covered the period from the turn of the 19th century to and throughout the 20th century. In the following we list some notes on Mach, Koyré and Kuhn to introduce the importance of the use of categories.

Ernst Waldfried Josef Wenzel Mach (1838–1916) generated a radically new vision of foundations of physics[1]. This new vision implied a deep criticism[2] toward Newton mechanics, particularly with regard to the metaphysical concepts of absolute *space* and *time*. Moreover, the concept of *cause–force* ultimately substituted the concept of *work* as the fundamental concept exploited by theoretical physics as well as the human mind to the extent it engages in related theorizing activity. Therefore, although mechanics is traditionally considered to be the basic theory in theoretical physics, thermodynamics should rather play this pivotal role. The contribution supplied by Mach can thus be considered especially relevant for the history of scientific thought. Indeed, once the birth of both thermodynamics and electromagnetism triggered the loss of the leading position of mechanics in physics, Mach's reflection was a pioneer attempt to understand the foundations of physics. Such reflection strongly influenced the theoretical activity carried by the

[*] It is a pleasure to thank prof. Antonino Drago for his precious collaboration with one of us and prof. Robert Zaborowski for last reading and suggestions.

[1] E. Mach, *The Science of Mechanics – A Critical and Historical Account of its Development* [1883], transl. T. J. McCormack, [4th ed.] Open Court – Merchant Book, La Salle 1974.

[2] E. Mach, *The Science of Mechanics* ... , chap. IV–V.

initiators of modern physics, namely Max Planck (1858–1947)[1] and Albert Einstein (1879–1955)[2]. So far we sketched a general picture of the intellectual environment which pushed for a new kind of history o science, beyond the cultural limits of scientists and philosophers of science at that time. Thenceforth this new kind of history of science came through either a mere collection of documents or a chronicle of certain remarkable events, the latter being the way in which history of science was mostly conceived during the 20th century. It also came through a simple editing of scientists' biographies as well a historical inquiry on the underlying concepts in physics, regarded as the invariable theoretical features of scientific events[3] in two thousands years.

Alexandre Koyré (1892–1964) brilliantly examined the birth of modern science by means of *ad hoc* historical categories. I shall therefore characterize this revolution, i.e. the birth of modern science, by two closely connected and even complementary features: (a) the destruction of the cosmos and therefore the disappearance from science – at least in principle, if not always in fact – of all considerations based on this concept, and (b) the geometrization of space, that is, the substitution of the homogeneous and abstract – however now considered as real – dimension space of the Euclidean geometry for the concrete and differentiated place–continuum of pre–Galilean Physics and Astronomy[4].

Table 1. Explaining Alexandre Koyré's choice for the history of science

The destruction of the cosmos, that is a replacement of the finite world, as it was hierarchically classified by Aristotle, with the infinite universe.
The geometrization of space, that is a replacement of Aristotle' physical (concrete) space with the abstract space of the Euclidean geometry.

Koyré underlined the logical and intellectual step ahead performed by just a few of the luminaries by that time, such as Galilei Galileo and Newton. Thanks to them science, *modern* by then, provided the basis for the next theories, so honourable for physicists and mathematicians. He considered above all that, among the intellectual factors, a basic role would have been played by the choice of the kind of infinite in mathematics[5]. Someone claimed about the same topic, remarking the artisans' work, considered, together with their inventions, the only responsible of the birth of the 17th century science. Koyré suggested the opposite thesis, according to which even laying upon a

[1] P. Cerreta & A. Drago, *La "Weltbild" di Planck reinterpretata col paradigma di Kuhn e col modello di teoria scientifica* in: F. Bevilacqua (ed.), *Proceedings of VIII SISFA Congress*, Napoli 1987, pp. 63–80.

[2] E.g. in theoretical physics on gravitation theories, Mach's principle or conjecture is the name given by Einstein to a vague hypothesis first supported by Ernst Mach.

[3] Excellent examples are: M. Jammer, *Concepts of Space: The History of Theories of Space in Physics*, Harvard University Press, Harvard 1954, M. Jammer, *Concepts of Force: A Study in the Foundations of Dynamics*, Harvard University Press, Harvard 1957, M. Jammer, *Concepts of Mass in Classical and Modern Physics*, Harvard University Press, Harvard 1961, M. Jammer, *Concepts of Mass in Contemporary Physics and Philosophy*, Princeton University Press, Princeton 1999, M. Jammer, *Concepts of Simultaneity: From Antiquity to Einstein and Beyond*, Hopkins University Press, Baltimore 2006.

[4] A. Koyré, *Newtonian studies*, Harvard University Press, Cambridge (Mass.) 1965, p. 53.

[5] AI = Actual Infinite, PI = Potential Infinite.

perfect artisan work the result will always be determined by the inaccuracy of measurements. In particular, he grasped that the birth of modern science cannot be explained just through the human works, but conceptual factors are needed, including discontinuity in history[1]:

> *The new science, we are told sometimes, is the science of craftsman and engineer, of the working, enterprising and calculating tradesman, in fact, the science of rising bourgeois classes of modern society. There is certainly some truth in these descriptions and explanations* […] *I do not see what the* scientia activa *has ever had to do with the development of the calculus, nor the rise of the bourgeoisie with that of the Copernican, or Keplerian, astronomy theories.* […] *I am convinced that the rise and the growth of experimental science is not the source but, on the contrary, the result of the new* theoretical, *that is, the new* metaphysical *approach to nature that forms the content of the scientific revolution of the seventeenth century, a content which we have to understand before we can attempt an explanation (whatever this may be) of its historical occurrence.*[2]

Thomas Kuhn (1922–1996) in *The Structure of Scientific Revolutions*[3] outlines a historical evolution of science as marked, every now and then, by *revolutions,* that is by *changing* a previous pattern. In brief, Kuhn based his research on the idea of scanning scientific structures in the history of science which can establish themselves as *paradigm* or produce a *replacement* of an old framework. However, today we know that his program was not completely adequate[4] to understand continuity and discontinuity and/or commensurability in the (historical) development of science. Unfortunately, and differently from the title of his book, he could not witness other revolutions than the birth of modern, that is Newtonian, science and the attempts on the crisis at the beginning of the 20th century. Such matters have already been underlined by some historians. Moreover, he used a unique Newtonian paradigm to analyze

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[1] A. Koyré, *From the Closed World to the Infinite Universe,* Johns Hopkins University Press, Baltimore 1957.

[2] A. Koyré, *Newtonian studies*, pp. 5–6.

[3] T. S. Kuhn, *The Structure of Scientific Revolutions*, University Chicago Press, Chicago 1962, T. S. Kuhn, *The function of Dogma in Scientific Research* in: *Scientific Change. Historical studies in the intellectual, social and technical conditions for scientific discovery and technical invention, from antiquity to present*, Heinemann Educational Books, London 1963, pp. 347–369, T. S. Kuhn, *Reflections on my Critics* in: *Proceedings of International Colloquium in the Philosophy of Science. Criticism and Growth of knowledge,* (eds) I. Lakatos & A. Musgrave, Cambridge University Press, Cambridge 1970, pp. 231–278, T. S. Kuhn, *Second Thoughts on Paradigms* in: *The Structure of Scientific Theories*, (ed.) F. Suppe, Illinois University Press, Urbana 1974, pp. 459–482, T. S. Kuhn, *Black–body theory and the quantum discontinuity, 1894–1912,* Oxford University Press, Oxford 1978, A. Koyré, *From the Closest World to the Infinite Universe,* A. Koyré, *Newtonian studies,* A. Koyré, *Du monde de «à–peu–près» à l'univers de la précision*, Armand Colin Librairie, Paris 1961.

[4] M. J. Klein, A. Shimony & T. J. Pinch, *Paradigm Lost? A Review Symposium* in: *Isis* 70, 1979, pp. 430–434, L. Kvasz, *On classification of scientific revolutions* in: *Journal for General Philosophy of Science* 30, 2/1999, pp. 201–232.

the development of theories: in this sense, the theories having different foundations as well as Renaissance statics and modern mechanics, Newtonian mechanics and Lagrangean one, chemistry[1] and thermodynamics were scarcely considered in his research.

On the other hand, beyond any question, the authority and authoritativeness of the Newtonian pattern survived almost unaltered and consistently until Pierre–Simon de Laplace (1749–1827) who had wisely widened the Newtonian theoretical model including short–rayed forces in order to apply it to the microscopic interaction, such as in the theory of capillarity. Following those mathematical speculations the traditionalist scientist Siméon–Denis Poisson (1781–1840) fully respecting[2] the Newtonian–Laplacean program interpreted all types of celestial and earthly phenomena through cause–forces. They typically are the central forces applied to other cases. One can see the modern laws of gases (e.g. Poisson) that they still valid today. To sum up, Kuhn's program, which was intended to explain all the scientific revolutions through the conceptual scheme of the Newtonian mechanics, found its path obstructed by the history of the evolution of *black body*[3]. As a matter of fact, when Kuhn took up this matter to explain the birth of quanta, in *Black Body Theory and the Quantum Discontinuity*, he had to give up the Newtonian paradigm that used to be his main inquiry category.

A recent criticism[4] connoted him with a *weak* historiographical–epistemological quality due to the Kuhn paradigm either in considering the *super–mechanical* elements of chemical theory as essential for the beginning of a revolution or in interpreting the birth of *quantum* that was no doubt a revolution in theoretical physics[5]. Moreover, when Kuhn introduced the Gestalt phenomenon, he was able to better justify the *replacement*, between a paradigm and other one by *continuity*/*discontinuity/incommensurability* items. But the latter produce a difficulty in communication between the two paradigms due to a *crucial variation of meaning* of the fundamentals concepts in the theories. Since the difficulties are so full of meaning, not all scientific results of previous paradigms are conserved. In this sense, after a crisis, a large and collected amount of scientific results are not ascertained. In the end, discovery seems recast in its scientific value whenever it has not undergone the filter of different approaches and scientific theories, even in conflict to

[1] R. Pisano, *A history of chemistry à la Koyré? Introduction and setting of an epistemological problem* in: *Khimiya Journal* 17, 2/2007, pp. 143–161.

[2] Lavoiser's biography: D. I. Duveen & H. S. Klickstein, *A bibliography of the works of A. L. Lavoisier*, Dawson & Sons, Ltd. & E. Weil, London 1954, J. Partington, *A History of Chemistry*, Macmillan, London 1964.

[3] A. Drago, *Storiografia del corpo nero: Rivisitazione e nuova impostazione* in: *Proceedings of XXV SISFA Congress* [available in pdf format via http://www.brera.unimi.it/SISFA/atti/atti2005.html], Milano, 2008, pp. C08.1–C08.6.

[4] See A. Drago & P. Cerreta, *Il programma storiografico di Kuhn caratterizzato secondo due programmi di ricerca sui fondamenti della scienza* in: A. Garuccio (ed.), *Atti XXIII Congresso Società Italiana Storia Fisica e Astronomia*, Bari 2003, pp. 120–130, M. J. Klein, A. Shimony & T. J. Pinch, *Paradigm Lost? A Review Symposium.*

[5] Fortunately the historical study of fundamentals and of changes of mind and theories allows to compare the scientific thought, specifically of physics and mathematics, with other fields of knowledge.

each other, since their foundation; so the evaluation itself of the scientific value of a theory cannot be an absolute one**.** It is enough to recall the interesting intellectual efforts proposed by Mach, Koyré and Kuhn.

2. What kind of history?

Traditional historiography of science identifies two types of history of science, resulting form investigation: (a) internal[1] historiographies, which tend to provide an explanation of history of science in terms of the variables belonging to the science itself or to the scientists' mind, (b) external historiographies, which tend to underline the decisive role played by social components through the development of science. This classification is clearly suggested by the specific nature of the subject inquired. Through 19th century, theories characterized by foundations different from the ones implied by the Newtonian paradigm, suggested that *subjective history* (e.g. history thought and experienced by scientists) has rather been different from *effective history* (e.g. history relying on the fundamental choices taken from scientists who drove interpretation of history by means of crucial choices). Similarly, *subjective history* has also been different from *objective history*, recorded on textbooks as a list of data together with the corresponding mathematical laws and taught by means of techniques and objective concepts. The latter also includes science taught by means of techniques and objective concepts.

Table 2. Some aspects of Objective, Subjective and Effective history

Objective History	**Subjective History**	**Effective History**
Historical facts, experiments, dates of discoveries	Foundations of scientific theories	Investigation by means of two logical–mathematical categories
Primary sources and early theories	Biography and correspondence	Logical organization of a scientific theory
Birth of new experimental apparatus. Academic context, societies, academies et al.	Concepts (e.g. intuitive and surrogate concepts). Mathematical content of a theory	The choice of kind of mathematical infinite used in a scientific theory. Changing infinite → change physical concepts

We focus on *effective history* that can suggest historiography categories for subjective history. The particular fundamental choice can generate variations of meaning for historical understanding. The special meanings are much less than one can imagine. They help understand by which fundamental concepts or mental categories the physicist/mathematician faced up and dealt with the *crisis in physics of 1900*. Let us remark that the use of other kinds of

[1] Koyré wrote notes upon this two types of historiography answering Henry Guerlac's talk: A. Koyré, *Etudes d'histoire de la pensée scientifique*, Gallimard, Paris 1973 [Engl. transl. in: A. C. Crombie, *Historical Studies in the intellectual, social and technical conditions for scientific discovery and technical Invention, from antiquity to the present* in: *Symposium on the History of Science, University of Oxford, 9–15 July 1961*, Heinemann, London 1963, pp. 847–857. The Henry Guerlac's relation in: *Symposium on the History of Science, University of Oxford, 9–15 July 1961*, pp. 797–817.

categories by means of logic and mathematics let the eventual *revolutionary* or *normal* logical character come out in a scientific theory or organization of theory further than his formal conception of infinite in the use of mathematics. The latter gives us a hint about the choice of formalism and continuous or discontinuous scientific progress[1].

3. Logics and mathematics: categories for inquiring

A new approach to history of foundations of science combines historical and epistemological aspects by means of logical and mathematical studies[2]. Nowadays it is called *historical epistemology* and mainly practiced by the Max Planck Institute for the History of Science of Berlin. Based on previous studies[3], here we add an historical investigation to explain specifically the organization of theories by means of their logical foundations. According to the historian of physics Antonino Drago[4], one can see that in the history of science we can encounter both logical[5] *axiomatically organised* theories[1] (AO

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[1] R. Pisano & I. Gaudiello, *The scientific approach in historical discourse* in: *Proceedings of ESHS 3rd Conference*, Austrian Academy of Science,Vienna 2009, pp. 187–197, R. Pisano, *On method in Galileo Galilei' mechanics* in: *Proceedings of ESHS 3rd Conference*, pp. 174–186, R. Pisano, *Continuity and discontinuity. On method in Leonardo da Vinci' mechanics* in: *Organon* 41, 2009 [in press].

[2] In July 2008 *The Max Planck Institute for the History of Science*, Berlin together with *The Center for Philosophy of Science*, University of Münster and *The Center for Logic, Philosophy and History of Science*, University of Rostock organized two international conferences: *Scientific knowledge in the context of thought style and paradigm – Ludwik Fleck and Thomas Kuhn on the development of scientific knowledge* and *What (Good) is Historical Epistemology?* Basic questions were: *What kind of historical enterprise is historical epistemology? Conversely, in what sense is it a form of epistemology?* [available: http://www.mpiwg–berlin. mpg.de/workshops/en/HistoricalEpistemology.html] One can also see works of *The Brazilian centre for logic, epistemology and the history of science* directed by Newton da Costa. Other works: J. Renn, P. Damerow & P. McLaughlin, *Aristotle, Archimedes, Euclid, and the Origin of Mechanics*: *The Perspective of Historical Epistemology*, Max Planck Institute for the history of science of Berlin, Berlin 2003, preprint n. 239, J. Renn, P. Damerow, G. Freudenthal, P. McLaughlin, *Exploring the limits of Preclassical Mechanics*, Springer, New York 1992.

[3] See G. Sarton, *A guide to the history of science*, Ronald Press, New York 1952, T. Nickles, *The Logic and Methodology of Science in Early Modern Thought* in: *Isis* 92, 4/2001, pp. 775–776, M. Bevir, *The Logic of the History of Ideas*, Cambridge University Press, Cambridge 1999, D. Batens & J. Meheus, *A Formal Logic for Abductive Reasoning* in: *Logic Journal of the IGPL* 14, 2006, pp. 221–236, E. Agazzi, *Logic and Methodology of empirical Sciences* in: *Modern Logic – A Survey*, E. Agazzi (ed.), Reidel, Dordrecht 1980, pp. 255–282, E. W. Beth, *Semantic of physical Theories* in: Freudenthal H. (ed.), *The concept & The Role of the Model in mathematics and natural and social sciences*, Reidel, Dordrecht 1961, M. L. Dalla Chiara, *Some Foundational Problems in Mathematics suggest by Physics* in: *Synthèse* 62, 1985, pp. 303–315, J. L. Destouches, *Physico–Logical Problems* in: *Studies in Logic and the Foundations of Mathematics. The Axiomatic Method with special references to geometry and physics*, L. Brouwer (ed.), North–Holland Publishing Co., Amsterdam 1959, pp. 390–405, J. L. Destouches, *Logique et Théorie Physique* in: *Synthèse* 16, 1966, pp. 66–73, J. L. Destouches, *Sur la Méchanique Classique et l'Intuitionnisme* in: *Indagations Mathematicae* 13, 1951, pp. 74–79, Y. Gauthier, *Internal Logic. A radically constructive logic for mathematics and physics* in: *Québec Studies in the Philosophy of Science*, M. Marion & R.S. Cohen (eds), Kluwer, Dordrecht 1995, pp. 107–122, E. Giannetto, *Fisica Quantistica e verità logica* in: *Epistemologia* 12, 1989, pp. 261–276, R. Giles, *Foundations for Quantum Mechanics* in: *Journal of Mathematical Physics* 11, 1970, pp. 2139–2151, R. Giles, *The Concept of a Proposition in Classical and Quantum Physics* in: *Studia Logica* 38, 1979, pp. 345–353, M. Przełęcki, *The Logic of Empirical Theories*, (ed.) G. B. Keene, vol. 10, Routledge & Kegan Paul, London 1969, J. Rothstein, *Information, Logic, and Physics* in: *Philosophy of Science* 23, 1/1956, pp. 31–35, C. F. von Weizsäcker, *Classical and quantum descriptions* in: J. Mehra (ed.), *The Physicist's Conception of Nature*, Reidel, Dordrecht 1973, pp. 635–667, A. Drago, *Le due opozioni*, La Merdiana, Molfetta 1991.

[4] A. Drago, *The process of induction as a non–classical logic's double negation*: *Evidence from classical scientific theories* in: *Mathware and Soft Computing* 3, 1996, pp. 295–308.

[5] In general, one can assume a non–classical logic as well as intuitionistic logic. See M. Dummett, *Principles of Intuitionism*, Clarendon Press, Oxford 1975, D. Prawitz & P. E. Melmnaas, *A survey of some con-*

theories) as well as those whose organisation requires non–axiomatic principles suggesting a method for solving a given problem for a theory which is thus logical *problematically organized* (PO theories). In brief, an AO theory is developed by *self–evident* principles and it is generally followed by the use of advanced mathematics (e.g. in Newton's theory). A PO theory is based on logic and methodological principles (in structural logic also called architectural[2]) which indicate a direction for the development of the theory. In addition, the lack of advanced mathematics is compensated by the use of double negation sentences (DNS) where the law of double negation $\neg\neg A \rightarrow A$[3] fails and much of the rest of the equation or theory is expressed by *Reductio ad absurdum* statements. In this type of theoretical organization, the scientific contents of DNS cannot be converted into an affirmative sentence corresponding to *A* because they lack scientific proof. This means that *A* cannot be directly stated: proposing that problem with the (idealistic) sentence *A* would mean cancelling problem *A* itself[4], e.g.

> [...] *the heavy bodies themselves* [have repugnance] to *the inverse motion, and never ever, in this state, they will not move, till then they will not be violently outcast by an external motor* […][5] [≠ *an external cause produces a motion*] […]. Simplicio. *I think it would begin at once, for having nothing to sustain it, its own weight could not help acting.* [≠ *a heavy projectile commences to drop immediately upon leaving the thing projecting it.*].[6]

In order to clarify the role of a DNS, for example, one can consider the principle of the impossibility of perpetual motion in mechanics which Simon Stevin (1548–1620) has already stated as: […] *ipsique globi ex sese continuum et aeternum motion efficient, quod est falsum*[7].

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nections between classical, intuitionistic and minimal logic in: A. Schmidt & H. Schuette (eds), *Contributions to Mathematical Logic*, North–Holland, Amsterdam 1968, pp. 215–229.

[1] AO: Axiomatical Organization in a theory, PO: Problematical Organization in a theory. See A. Drago, *Le due opozioni*.

[2] The term *methodological* used in this paper recalls Sadi Carnot's theory based on special principles like impossibility of a perpetual motion, which are different from those axiomatic used by Aristotle, or used in the scientific theories, e.g. Newton's mechanics. In this work, from now onwards, I will use the *Principle of theory*. The theme is also developed in a recent work: D. Capecchi & R. Pisano, *La teoria dei baricentri di Torricelli come fondamento della statica* in: *Physis* 44, 1/2007, pp. 1–29.

[3] A sentence, its negative $\neg A$, and its double negation $\neg\neg A$ in mathematical logic: *non non A follow A*.

[4] In the following paper are listed Carnot's DNSs: A. Drago & R. Pisano, *Interpretation and reconstruction of Sadi Carnot's* Réflexions *through original sentences belonging to non–classical logic* in: *Fondazione Giorgio Ronchi* 59, 5/2004, pp. 615–644.

[5] See *Delle macchie solari* in: *Le Opere di Galileo Galilei*, vol. 5, (ed.) A. Favaro, G. Barbera Editore, Firenze 1895, p. 134 [English transl. is ours – R. P. & I. G.].

[6] G. Galilei, *Dialogues concerning the two chief world systems* [1632], transl. S. Drake, The University of California Press, Berkeley 1967, p. 194.

[7] It is not true (*falsum*) that the globe moves by itself and has not end (*aeternum*). (S. Stevin, *Liber primus Staticae. De staticae elmentis* in: *Tomus quartus mathematicorum hypomnematum de statica*, Lugodini Batavorum 1605 (*post.* 1608), p. 35). See also E. J. Dijksterhuis, *The principal works of Simon Stevin*, vol. 1, N.

The corresponding affirmative sentence *motion has an end* is empirically doubtful because infinite observation time is needed to verify it or a way to verify that friction will not eventually and definitively vanish. Consequently, the principle of the impossibility of a perpetual motion is better expressed by means of a DNS. Let us notice that this principle cannot play the role of *a priori* sentence, e.g. evident to reason, but instead comes from common experience. In this sense a particular case–study belongs to non–classical logic.

A clarification of its semantic is very important for the understanding epistemological objectives as Condillac Etienne de Bonnot (1714–1780) in his work *La Logique*[1] already emphasized the importance of language in logical reasoning, stressing the need for a scientific language for as scientific theories in the 18th century. According to Antonino Drago, theoretical choices concerning the two options – AO or PO – allow a proper detection of the foundations of the scientific setting of a theory and choices of its kind of infinite in mathematics. Such theoretical choices let also emerge the foundational differences between different theories.

Table 3.
Historical approach according to four Drago models based on two choices[2]

Mathematics	**Aristotelian – Axiomatic Organization (AO)**	**Problematical Organization (PO)**
***Actual Infinite* (AI)**	**Newtonian model** 1687 Mechanical Newtonian 1700 Mechanical nature of optics 1862 Maxwellian electromagnetism	**Lagrangean model** 1788 Lagrangean mechanics (and mechanics by Maupertuis, mechanics of variations) 1890 Statistical mechanics
***Potential Infinite* (PI)**	**Cartesian model** 1630 Geometrical optics 1803 (*Principes*) mechanics by Lazare Carnot 1848–1851 Mechanical theory of heat 1878 Chemistry–physics	**Carnotian model** 1783 (*Essai*) Mechanics by L. Carnot 1824 Thermodynamics by Sadi Carnot 1866 Classical chemistry

V. Swets & Zeitlinger, Lisse 1955, pp. 174–179, p. 507 & p. 509, M. Clagett & E. Moody, *The medieval science of weights (Scentia de ponderibus)*, The University of Wisconsin Press, Madison 1960 [Italian transl. L. Sosio: *La Scienza meccanica nel Medioevo*, Feltrinelli, Milano 1981, p. 123, n. 54].

[1] E. B. Condillac, *La logique par Condillac* [1780], Verdier Quai Des Augustins, Paris 1821, pp. 222–225. See also E. B. Condillac, *La Langue des calculs* [1798]. Let us note that Antoine Laurent de Lavosier also wrote about the role played by logic and language in science. He started the *Preface* of his revolutionary *Traité élémentaire de Chimie* [1789], p. XXV by saying: *Il* [Condillac] *y établit que nous ne pensons qu'avec le secours des mots* [...] *enfin que l'art de raisonner se réduit à une langue bien faite*. See also pp. XXVI–XXXVIII.

[2] Adapted by A. Drago, A. Drago, *Le due opozioni*. See also A. Drago & R. Pisano, *La novità del rapporto fisica–matematica nelle* Réflexions *di Sadi Carnot* in: *Fondazione Giorgio Ronchi* 62, 4/2007, pp. 497–525.

4. Case–studies: an excursus

4.1. On Archimedes, Galilei and Toriccelli's mechanics

It has often been assumed that Newtonian mechanics was capable of encompassing – without any consequences – every theoretical and deductive formulation of the 18th and 19th centuries (Lazare Carnot, Lagrange et al. ...). Such assumptions led to the conclusion that the various formulations of mechanics can be regarded as equivalent. As a result, foundations of theoretical physics were widely obscured. But

> [p]*urely mechanical phenomena do not exist* […] *On the other band, thermal, magnetic, electrical, and chemical conditions also can produce motions. Purely mechanical phenomena, accordingly, are abstractions, made, either intentionally or from necessity, for facilitating our comprehension of things. The same thing is true of the other classes of physical phenomena.* [...] *The view that makes mechanics the basis of the remaining branches of physics, and explains all physical phenomena by mechanical ideas, is in our judgment a prejudice. Knowledge which is historically first, is not necessarily the foundation of all that is subsequently gained. As more and more facts are discovered and classified, entirely new ideas formed. We have no means of knowing, as yet, which of the physical phenomena* go deepest, *whether the mechanical phenomena are perhaps not the most superficial of all, or whether all do not go* equally deep. *Even in mechanics we no longer regard the oldest law, the law, of the lever, as the foundation of all the ether principles. The mechanical theory of nature, is, undoubtedly, in an historical view, both intelligible and pardonable; and it may also, for a time, have been of much value. But, upon the whole, it is an artificial conception. Faithful adherence to the method that led the greatest investigators of nature, Galileo, Newton, Sadi Carnot, Faraday, and J. R. Maver, to their great results, restricts physics to the expression of* actual facts, *and forbids the construction of hypotheses behind the facts, where nothing tangible and verifiable is found. If this is done, only the simple connection of the motions of masses, of changes of temperature, of changes in the values of the potential function, of chemical changes, and so forth is to be ascertained, and nothing is to be imagined along with these elements except the physical attributes or characteristics directly or*

indirectly given by observation.[1]

Let us consider, for instance, that since Newtonian principles refer to a single particle, they cannot deal with extended bodies systems, which include bonds rather than energetic matter without adopting in the collision theory. In the following we present a recent inquiry[2] on the scientific organization and eventual explanation of continuity or discontinuity in mechanics according to three crucial scientists, Archimedes, Galilei and Torricelli**:**

Table 4. Epistemological inquiry on Archimedes, Galilei and Torricelli

– Characteristic concepts – Philosophical item	**Archimedes (287–212 BC)**	**Galileo Galilei (1564–1642)**	**Evangelista Torricelli (1608–1647)**
Cultural value of the theory	Theoretical and experimental	Theoretical and experimental. Also philosophical?	Theoretical
Organization of the theory	– PO (mechanics) – AO (geometry)	– PO (mechanics) – AO (geometry)	– PO (mechanics) – AO (geometry)
Definition of bodies system	– As sets together and constituting one body – Its dimension is greater – Without explaining its type of connection	– *Aggregati*: – sets together and constituting one body – Its dimension is greater – Without explaining its type of connection	– *Congiunti*: – sets together and constituent one body – Tied up way or untied – Body of dimension greater – Explaining its type of connection
Foundational concept of static theory	– Centre of gravity – Some Aristotelian ideas	Centre of gravity of Archimedes Weights are really geometrical figures–magnitudes intended as masses of a balance;	Centre of gravity of Archimedes Weights are really geometrical figures–magnitudes intended as masses of a balance;

[1] E. Mach, *The relationship of mechanics to physics* in: *The Science of Mechanics*, p. 495.

[2] R. Pisano, *Il ruolo della scienza meccanica nella progettazione degli architetti e degli ingegneri del Rinascimento*, Ph. D. dissertation from University of Roma “La Sapienza”, 2008, vol. 1, pp. 116–134 [A pdf of vol. 1 & vol. 2 are available via *International Galilean Bibliography, Istituto e Museo di Storia delle Scienze*, Firenze: http://biblioteca.imss.fi.it/], R. Pisano, *Brief history of centre of gravity theory. Epistemological notes* in: *Proceedings of 2nd ESHS Congress*, Polish Academy of Arts and Science, Kraków 2007, pp. 934–941, R. Pisano, *Il ruolo della scienza archimedea nei lavori di meccanica di Galilei e di Torricelli* in: *Da Archimede a Majorana: La fisica nel suo divenire*. *Proceedings of XXVI SISFA Congress*, E. Giannetto, G. Giannini, D. Capecchi, R. Pisano (eds), Guaraldi Editore, Rimini 2009, pp. 65–74, D. Capecchi & R. Pisano, *La meccanica in Italia nei primi anni del Cinquecento. Il contributo di Niccolò Tartaglia* in: *Proceedings of XXV SISFA Congress* [available in pdf format via: http://www.brera.unimi.it/SISFA/atti/atti2005.html, Milano, 2008, pp. C17.1–C17.6, D. Capecchi & R. Pisano, *La teoria dei baricentri di Torricelli come fondamento della statica*.

		Geometrical figures–magnitudes to which one also attributed numerical values; Geometrical demonstration of statical theorems; Geometrical form implicit in weightless beams; Indirect reference in geometrical form to the Law of the Lever; Substituting for a segment of a material beam as point of a weightless segment; Using *ad absurdum* proofs; The parallelism of the forces(–weight')s directions and masses are attracted toward the centre of the Earth	Geometrical figures–magnitudes to which one also attributed numerical values; Geometrical demonstration of statical theorems; Geometrical form implicit in weightless beams; Indirect reference in geometrical form to the Law of the Lever; Substituting for a segment of a material beam as point of a weightless segment; Using *ad absurdum* proofs; The parallelism of the forces(–weight')s directions and masses are attracted toward the centre of the Earth
Scientific paradigms	*Problemata mechanica*	– *Problemata mechanica* – Archimedes' Law of the lever – Principle of virtual works – Hero's machines	– *Problemata mechanica* – Archimedes' Law of the lever – Principle of virtual works – Hero's machines
Type of infinite in mathematics	– PI (Potential Infinite) – → AI (Actual Infinite)	– PI – → AI	– PI – → AI

Central problem of the theory	In order to establish criteria to determinate the centre of gravity for single and composed geometrical bodies (*On the Equilibrium of planes*)	In order to determinate the centre of gravity for single and composed geometrical bodies by means of Archimedes' criteria. (*Discorsi e Dimostrazioni Matematiche)*	Re–visiting Galilei's ballistic theory by means of Archimedean equilibrium theory (*Opera Geometrica*)
Techniques of arguing	*Reductio ad Absurdum*	*Reductio ad Absurdum*	*Reductio ad Absurdum*
Techniques of calculus	Method of exhaustion	Archimedes' method of exhaustion	– Archimedes' method of exhaustion – (Cavalieri's method)
Solutions	Applications of his criteria to single bodies. Generalization for comparison	– Applications of Archimedean criteria to equilibrium of bodies *aggregate*. – Applications to mechanical theory, elasticity theory and fortifications – architectural theory	Applications of Archimedean criteria to equilibrium of bodies *congiunti*

In the following we present table 5 which reassume a hypothesis of the scientific traditions based on their foundations and classified by two choices, AO and PO. To be brief, we do not exhaustively comment them[1].

Table 5. Different choices for mechanical traditions view

Engineering Mechanical Traditions	**1° *classical* Mechanical Traditions**	**2° *classical* Mechanical Traditions**
Tartaglia	Galilei	Galilei
Galilei	Descartes	Huygens
Bernoulli	Newton	Leibniz
Borda	Euler	D'Alembert
Carnot & Lazare	Laplace	Lagrange
PO	**AO**	**PO**

[1] See R. Pisano, *Il ruolo della scienza meccanica* ... .

A global view of tables 3, 4 and 5 should establish the different foundations of the mechanics. The mechanics of Newton are purely geometrical. He deduces his theorems from his initial assumptions (AO principles) entirely by means of geometrical constructions[1], e.g. Newton by means of AI, AO and the absence of collision theory and its consequences is different from Lazare Carnot's mechanics. In fact, the latter included collision theory and so quantity of motion interpreted by non–infinitesimal mathematics (PI) based on a problematic organization of the theory (PO). Lagrange's formulation is interesting. It can be set into an intermediate stage, due to his option for an AI mathematics, which indeed is meant to contribute to the search of a new mathematical technique suitable for any mechanical problem. To the discontinuity presented by a different organization of theory (table 5), we add the choice (table 6) for the kind of infinitum in mathematics, AI and PI:

Table 6. Discontinuity in the history of science by means of crucial choices[2]

Main date	**1630 ...**	**1687 ...**	**1782 ...**	**1870 ...**	**1905 ...**
AO Theories	AO Geometrical Optics	AO Newtonian Mechanics → Acoustical Optics	AO Newtonian Mechanics → Optics, Acoustics	AO Newtonian Mechanics Thermodynamics Electromagnetism	AO Physics–Mathematics Electromagnetism Thermodynamics (Caratheodory) Statistical Mechanics
PO Theories	?	?	PO Mechanics (Torricelli, L. Carnot and Lagrange) Chemistry & Thermodynamics (S. Carnot)	PO Chemistry (Lavoisier, Dalton)	PO Special relativity Quantum Mechanics (AO & PO)
Paradigm: Continuity or Discontinuity	Mechanics (Archimedes) & Euclidian Geometry	Mechanics	Mechanics vs Chemistry	Mechanics vs Thermodynamics	Statistical Mechanics & Relativity & Electromagnetisms
Crucial Items	Luminous ray	Absolute space & time & Cause–Force	Absolute space & time Cause–Force vs. Matter, work, bounded space & time	Force, field, ether vs Entropy	Particles & wave vs ∞ Particles

It should be pointed out, after a long–lasting setting out of various choices, how mathematics reached a couple of strong choices – AI and AO – with the establishment of set theory at the end of the 19th century. Furthermore a relevant concept should be highlighted: the *incommensurability of different*

[1] E. Mach, *The Science of Mechanics*, p. 465.

[2] Adapted by A. Drago, *Le due opozioni*.

theories. This concept was intuitively introduced by a historian and a philosopher of science, namely Th. S. Kuhn[1] and P. K. Feyerabend.[2] Their definition of this concept is rather approximate and based on few historical, e.g. Newtonian mechanics in relation to special relativity or quantum mechanics.

4.2. On Lavoisier's chemistry

In general, the first scientific theory[3] assumed as systematic and mathematical was René Descartes' (1596–1650) theory put forward in his *Optics*[4]: any phenomenon was followed by its mathematical interpretation, eventually enriched with a geometrical one. Later on, much relevance was attributed to the birth of the Newtonian mechanics since its mathematical content seemed full of potential and Newton's project so all inclusive as to involve any other theories (optics as well) through an arrangement based on the Aristotelian–axiomatic model (AO). It is well known that Isaac Newton (1642–1727) would not publish his works upon optics (published only in 1730). He was quite deluded about that since he had not been able to circumscribe the whole of the phenomena within axiomatic. As a matter of fact, the English scientist considered that arrangement of major importance. Newton did a lot of research in the field of chemistry as well, though once again he did not manage to produce an Aristotelian–axiomatic theory. As a conclusion of the *Optiks*[5] (1704), he formulated 31 long Queries by which the unsolved problems and his doubts about the theory were expressed. He dealt much with Chemistry, particularly in *Query 31*. More specifically, he argued about matters previously anticipated within a treaty on acids and rigid bodies related to the gravity attraction force, stating a definition of acids

> *as endowed with a huge Attraction Force; their Activity consists of this Force*[6]. *In particular, about hard bodies he observed.*
>
> *The parts of all homogeneal hard bodies which fully touch one another, stick together very strongly. And for explaining how this may be, some have invented hooked Atoms, which is begging the Questions; are glued together by rest, that is, by an*

[1] T. S. Kuhn, *The Structure of Scientific Revolutions*, T. S. Kuhn, *Reflections on my Critics*.

[2] K. P. Feyerabend, *Against Method*, Verso Books Paperback, Humanities Press, New York 1989.

[3] Obviously I do not exclude the ancient and Renaissance (embryonic–scientific) theories: i.e., studies on *the centre of gravity*.

[4] The essay on *Optics* is part of his most famous work *Le Monde*, in which he deals with his mechanicist and rational observations, i.e. *La Dioptrique* (1634), *Les Météores* (1635) and *La Géométrie* (1636) in: R. Descartes, *Discours de la méthode* (March 1637). In these works, a particular study about some natural phenomena appears (i.e light) connected with the human senses and profound mathematical interpretation follows.

[5] The *31* Queries in *Optiks* were Newton's last work and they were thought about and delayed in publishing for long. See I. Newton, *Queries* in: *Optiks*, [4th English Edition corrected] Innys, London 1730.

[6] Gravitational – it should be clarified that Newton is talking about the gravitational force and not about any dynamic force such as he presented it in the second law. This is important because both laws will be (in the next century and in subsequent historical investigations) objects of discussion, but in different way one from each other. My aim is to describe the role played by g (acceleration of gravity) and not a (general acceleration).

> *occult Quality, or rather by nothing* [...] *And therefore hardness may be reckon'd the Property of all uncoumpounded Matter. At least, this seems to be as evident as the universal Impenetrability of Matter. For all bodies, so far as Experience reaches, are either hard, or may be harden'd; and we have no other Evidence of universal Impenetrability, besides a large Experience without an experimental Exception. Now if compound Bodies are so very hard as we find some of them to be, and yet are very porous, and consist of Parts which are void of Pores, and were never yet divided, must be much harder*.[1]

The history of the classical chemistry is characterized by two burning aspects. In 1970, Arnold Thackray in *Atoms and Power* introduced a history of the birth of classical Chemistry, characterizing it according the above two basic aspects[2]. The first one concerns the intellectual and fundamental contrast between Lavoisier's new theory and the prevailing view conceiving of scientific theory as well as typically considered in a Newtonian context. The second aspect is correlated with John Dalton's (1766–1844) two essential choices: an organization evidently problematic of the theory and a mathematics with the only use of the potential infinite, that is to say, the study of the solution to the problem of the atomic weights through a kind of mathematics discriminating the matter. According to those really bold choices the British physicist and chemist built up his new concept of the world. The title of his famous work *A New System of Chemical Philosophy* (1808) already suggested an intellectual revolution. In *Atoms and Power*, Thackray clearly expresses his categories of historical interpretation:

> *The theory*[3] *has two essential components–belief in the inertial homogeneity of all matter and its possession of an* internal structure*, and acceptance of attractive and repulsive forces as proper categories of*[4] *explanation*[5] [...] *A third and more ambiguous Newtonian category, the ether, thought often referred to or hinted at, did not feature prominently before 1740's*.[6]

The *inertial homogeneity of matter*, quoted by Thackray, is referred to the Newtonian conviction of a matter hierarchically ordered and strictly

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[1] I. Newton, *Queries* in: *Optiks*, pp. 388–389.

[2] See R. Pisano, *A history of chemistry à la Koyré? ...* .

[3] Newtonian chemistry – it is understood that Thackray is talking here about the early chemical ideas of Newton upon affinity theory of.

[4] interpretation.

[5] historical.

[6] A. Thackray, *Atoms and Powers. An Essay on Newtonian Matter and the development of Chemistry*, The Harvard University Press, Cambridge (Mass.) 1970, p. 122.

structured. Whereas the second category, the admission of *short–rayed forces*, is referred to the fact that, according to such a view of science, for chemistry as well as for the celestial and earthly mechanics, a quantifying method is necessary. This goes through the measurement of those cause–forces (dynamic forces as infinitesimal object) which are typical of Newtonian theory. As regards Chemistry and the measurement of short range forces, the theory of the chemical affinities should be considered. Of course, in Newtonian mathematics, these forces include the differential equations. By those categories, Thackray interpreted the application of the Newtonian theory to pre–Lavoisier Chemistry. Thackray's categories well suit the study of Newtonian chemistry. Moreover they can show very well the difference of that theory from Lavoisier and Dalton theories. He ends up his book this way: *The*[1] *theory was profoundly antiphysicalist and antiNewtonian*[2] *in its rejection of the unity of matter, and its dismissal rejected the short–rayed forces.*[3]

In 1789 chemistry produced a real revolution and Antoine Laurent Lavoisier (1743–1794)[4], as well as the chemists of his time, searched for the basic principles of this new theory in a revolutionary fashion[5]. Nevertheless those *principles* were not the same as in Newtonian mechanics: neither were they a self–evident property of truth in the Aristotelian sense. Moreover Lavoisier's revolution started with the rejection of the traditional system of principles of the four elements. Let us think of the dissociation of water in *H* and *O* which was particularly a matter of contrast with the old Aristotelian theory and enabled Lavoisier, e.g., to start a *battle* against the *phlogiston theory* as an explanation of phenomenon of the fire. He replaced it with the combination with oxygen and on the whole two new elements: *calorique et lumière*[6]. This new way of considering science appeared similar to a *mental illusion*, that is like the impossibility of actually theorizing. This is because, according to scientists of the time, the lack of *real principles* made it impossible the process of making theory out of a mathematical model and consequently the building up of what was then considered a *true theory* (i.e. *à la* Newton). Later, Dalton's contribution[7] to the matter was crucial. Thanks to him chemical science was no more a theory simply opposite to the AO. The real *discontinuity* in Dalton's book consisted of dealing but with an only problem, either in the form of a program:

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[1] chemical – should be added as Thackray is talking about early chemical ideas of Newton of affinity theory.

[2] such as a kind of discontinuity.

[3] A. Thackray, *Atoms and Powers ...* , p. 279.

[4] A. L. Lavoisier, *Traité élémentaire de Chimie*, Gauthier–Villars, Paris 1789. In this *traité* he also dealt with the language in science and chemistry particularly, citing the *Logique* (1780) by Etienne Bonnot de Condillac. See also A. L. Lavoisier, *Mémoires sur la Chaleur* in: *Histoire de l'Académie Royale des Sciences*, Paris [1780 and first reading on 18th June 1783] 1784, pp. 355–408.

[5] Some years before, with Claude–Louis Berthollet (1749–1822), Antoine–François de Fourcroy (1755–1809) and Louis–Bernard Guyton de Morveau (1737–1816) Lavoisier published *Méthode de nomenclature chimique* (1787).

[6] A. L. Lavoisier, *Traité élémentaire de Chimie*, pp. 34–36.

[7] J. Dalton, *A New System of Chemical Philosophy*, Russell & Allen for R. Bickerstaff, Manchester – London 1808.

> *By elementary principles, or simple bodies, we mean such as have not been decomposed, but are found to enter into combination with other bodies. We do not know that any one of the bodies denominated elementary, is absolutely in–decomposable, but it ought to be called simple, till it can be analyzed.*[1]

Coherently with the individuation of this central problematical view of chemistry, Dalton goes on pointing out a method, either an ideal one, to combine elements among themselves. Such a singular method for the science at that time consists of a clearing illustration and by the well–known series of the seven rules[2]. He suggested atoms combining only in the simplest forms. In order to apply his rules Dalton used, more than a mathematical device, some models made of wood of the combination of the atoms: *When an element A has an affinity to another B, I see no mechanical reason why it should not take as many atoms of B as are presented to it.*[3] Actually the Newtonian mass is above all an inertial one, while Lavoisier's mass is gravitational.

Table 7. Discontinuity in foundations of homogeneous theoretical fields: Newtonian chemistry and Lavoisieran chemistry[4]

Burning items of the theory	**(Mechanical nature) Newtonian chemistry (AO, AI)**	**Lavoisieran chemistry (PO, PI)**
Space	Infinite and absolute	Assumed as volume on the whole
Time	Absolute	Assumed as a measure to mark a before and an after; with regard to the rate reactions)
Atom	Infinitesimal part of matter	Plurality of elements
Fluid	Phlogiston (corporeal)	Caloric (incorporeal)
Mass	Inertial	Gravitational
Interaction	Force–cause	Reaction and balance
Problem of the theory	Nature of the matter Molecular theory: attractive and repulsive forces	Indivisibility *Chemical affinities* theory through the accomplishment of the nomenclature and chemical elements

[1] J. Dalton, *A New System of Chemical Philosophy*, pp. 221–222.

[2] The seven rules concerned combinations and weights that were to represent the quantitative foundation of the modern Chemistry.

[3] J. Dalton, *Inquiry Concerning the Signification of the Word* in: J. Dalton, *Particle as used by Modern Chemical Writers* in: *J. Nicholson's Journal* 29, 1811, pp. 143–151.

[4] Adapted by A. Drago, *Le due opozioni*.

Arguing techniques	Differential equations	Arguing *by absurdum* proof and elementary mathematics
Solutions	Any possible solution, for a given force, from –∞ to +∞	Saturation degrees of oxygen; variation of some acids names endings

Those variations in meaning are so many that chemists can choose to use the word *principle* as well, typical of Newtonian theory and of the AO theories, though with a completely different meaning. Therefore, here more than usual, the problem arises whether two non–measurable theories are incompatible, too. As a matter of fact, the history of chemistry proves its historical incomparability to the point that physicists (busy as mechanicists) still underestimate completely the classical chemistry as the true theory.

4.3. On Sadi Carnot's theory of heat

Around 1824 the theory of heat, at least in France, remained the rising mechanical theory of heat[1], and Sadi Carnot's (1796–1832) *Réflexions sur la Puissance Motrice du Feu*[2] was reviewed by the *Academy of Sciences* thanks to a commemorative essay written by Sadi's friend Emile Clapeyron[3] (1799–1864) in 1834. Subsequently, *Réflexions* was almost universally ignored for 25 years. Sadi Carnot's friends, students at the *Ecole Polytechnique de Paris*, considered it a *difficult book*[4]. The work does indeed contain some surprising innovations: the idea of cycle, an upper bound to efficiency[5] of heat machines, a *reductio ad absurdum* theorem proof, new laws of gases. But its

[1] R. J. C. Clausius, *Ueber die bewegende Kraft der Waerme und die Gesetze* in: *Poggendorffs Annalen der Physik und Chemie* 155, 1850, pp. 368–397 & pp. 500–524 [English transl. in: E. Mendoza, *Reflections on the Motive Power of Heat Engines*, Dover, New York 1960, pp. 73–74 & pp. 109–152], W. Thomson, *On an absolute thermometric scale founded on Carnot's theory of the motive power of heat, and calculated from Regnault's observations* in: *Cambridge Philosophical Society Proceedings* 1, 5/1848, pp. 66–71. See also: W. Thomson, *On the dynamical theory of heat* in: *Mathematical and Physical Papers*, vol. 1, Cambridge University Press, Cambridge 1851, pp. 175–183. More recently: R. Pisano & D. Capecchi, *La Théorie Analytique de la Chaleur. Notes on Fourier and Lamé* in: *Gabriel Lamé, les pérégrinations d'un ingénieur du XIX*e *siècle*, (ed.) E. Barbin, *Bulletin de la Sabix* 44, 2009, pp. 83–90.

[2] S. Carnot, *Réflexions sur la Puissance Motrice du Feu sur les machinés propre à développer cette puissance* [1824], édition critique par Fox Robert, J. Vrin, Paris 1978. See also A. Taton (ed.), *Sadi Carnot et l'essor de la thermodynamique*, Éditions du Centre National de la Recherche Scientifique – École Polytechnique, Paris 1976.

[3] Clapeyron wrote *Mémoire sur la Puissance Motrice du Feu* in: *Journal de l'Ecole Polytechnique* 14, 1834, pp. 153–191. The work was mathematically different from S. Carnot's book and the famous diagram *PV* (for representing the Carnot's cycle) is, for the first time, introduced.

[4] A. Drago & R. Pisano, *La nota matematica nelle* Réflexions sur la Puissance motrice du feu *di Sadi Carnot: interpretazione del calcolo con il metodo sintetico* in: *Quaderni di Storia della Fisica – Giornale di Fisica* 13, 2005, pp. 37–58, R. Pisano, *L'interpretazione della nota matematica nelle* Réflexions sur la Puissance Motrice du Feu (1824) *di S. Carnot* in: *Proceedings of XX SISFA Congress*, Bibliopolis, Napoli 2001, pp. 205–230.

[5] T. S. Kuhn, *Sadi Carnot and the Cagnard Engine* in: *Isis* 52, 1961, pp. 567–574, T. S. Kuhn, *Carnot's version of cycle* in: *American Journal of Physics* 23, 1955, pp. 91–94, T. S. Kuhn, *Engineering precedent for the work of Sadi Carnot* in: *Proceedings of IX Congrès international d'Histoire des sciences*, Barcelone – Madrid 1959, Barcelona – Paris 1960, pp. 530–535. See also T. S. Kuhn, *Engineering precedent for the work of Sadi Carnot* in: *Archives Internationales d'Histoire des Sciences* 13, 1960, pp. 251–255.

interpretation is difficult also because, surprisingly, Sadi Carnot obtained results that were almost all exact even though based on erroneous heat conservation (caloric theory). On the other hand [h]*owever, the novelty of the* Réflexions *was of a kind that it was difficult to appreciate, so much so that the book probably bred incomprehension rather than excitement among the few contemporaries who read it.*[1]

In order to understand what appears to be a logical difficulty in Sadi Carnot's book, we need to understand what was *naturally* understandable for scientists at that time: the Newtonian paradigm in physics and mathematics. The following table 8 synthetically expresses the novelty of the fundamental concepts of Sadi Carnot's theory, in contrast to Newton's theory which prevailed at that time.

Table 8. A discontinuity in foundations of different theoretical fields: Newton's mechanics and S. Carnot's theory of heat

Fundamentals concepts	**Isaac Newton (1642–1727) Mechanics (AO, AI)**	**Sadi Carnot (1796–1832) Thermodynamics (PO, PI)**
Space	Infinite and absolute	Bounded–relational
Time	Absolute	Finite variation in time
Inertia	As perpetual	Impossibility of a perpetual motion
Basic–concept	Acceleration	Transformation
Interaction	Force–cause	Work
Mathematical problem	$F = ma$	dq/t Integration
Reasoning technique	Differential equations	Cycle
Solution	All kin of motion for a force from $t = -\infty$ to $t = +\infty$	Maximum of efficiency of heat machines

The absence of absolute space and time, as essential indexes on which the Newtonian theory is founded and the temporal variations of physical magnitudes are replaced by reasoning, through the new concept of cycle. The theory contained in *Réflexions* seemed therefore revolutionary compared to previous theories. Recent works[2] show that Sadi Carnot's thermodynamic theory was a PO theory, based upon problems of the validity of caloric[3] theory and around the calculation of the maximum efficiency[4] of a heat machine.

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[1] S. Carnot, *Introduction* to: Réflexions *on the Motive Power of Fire*, a critical edition with the surviving manuscripts, transl. R. Fox, Manchester University Press, Manchester 1986, p. 22.

[2] A. Drago & R. Pisano, *Interpretation and reconstruction of Sadi Carnot's* Réflexions ... , A. Drago & R. Pisano, *La novità del rapporto fisica–matematica nelle* Réflexions ... , A. Drago & R. Pisano, *S. Carnot's* Réflexions*: a theory based on non–classical Logic* in: *The Bulletin Symbolic Logic* 8, 2002, pp. 130–131.

[3] For an interpretation of caloric from historical suggestive and surrealistic points of view: H. U. Fuchs, *A surrealistic tale of electricity* in: *American Journal of Physics* 54, 1986, pp. 907–909.

[4] R. Fox, *Watt's expansive principle in the work of Sadi Carnot and Nicolas Clément* in: *Notes and records of the Royal Society of London* 24, 1970, pp. 233–253.

5. Final remarks: continuity and discontinuity in the foundations?

An approach to history of foundations of science that combines historical and epistemological aspects by means of logical and mathematical inquiry is possible. This kind of approach moves to a meta–theory of history of physics based on its foundations. The investigation by categories highlights the kind of foundations for studying an alternative to the Newtonian and the idealistic paradigms: Leibniz, D'Alembert, Lazare Carnot, Sadi Carnot's thermodynamics, Faraday's electric theory. In view of the matters brought up in result of this study and in Drago's historical approach, two scientific theories can be defined as discontinuous with regard to their logic and mathematical foundations whenever they are:

1) systematically (structurally and semantically) organized,
2) mathematized,
3) different at least by one of the two crucial choices of foundations.

The sustained *life* of the dominant model among the 4 presented ones (table 3), suggests a continuity in foundations, and by extension it can be considered such as a *normal* Kuhnan concept. Discontinuity in logical and mathematical foundations based on these models suggests a drastic variation of the dominant model. This aspect brings to mind the revolutionary thought that, more generally, they can be considered as *revolutions* of the Kuhnan concept. An overview of choices made by some historians is presented in table 9:

Table 9. Historians and their crucial choices

Historian	**Key word**	**Categories**	**Main Subject**
Mach (1838–1916)	Foundations	Economy of thought	Mechanics and heat
Duhem (1861–1916)	Continuity	Geometry, infinite, cosmos, active	Statics
Koyré (1892–1964)	Discontinuity	*The destruction of the cosmos and geometrization of space*	Mechanics
Crombie (1916–1996)[1]	Regularity	Individual regularity and regularity of population	Mechanics
Kuhn (1922–1996)	Paradigm	Normal, anomaly	Classical physics
Drago (1938–)	Foundations	Organization of a scientific theory and choice of mathematical infinite in theory	Classical and modern physics

[1] *Six major categories–headings*: 1. Arguing by means of analyses and synthesis (postulation). 2 Exploration by means of controlled experiments, observation and measure. 3 The construction of hypothetical modeling. 4. The taxonomy 5. The method of historical derivation (genetic method) was applied first to languages and human cultures, then to geological history (evolution). 6. Probabilistic and statistical analysis (A. C. Crombie, *Styles of Scientific Thinking in the European Tradition*, Duckworth, London 1994).

		Surrogates concepts: *the evanescence of force–cause and discretization of matter*	and mathematics. Classical chemistry and thermodynamics
Thackray (1939–)	Foundations	Inertial homogeneity of matter and short–range forces	Classical chemistry–physics

In the end, we should remark that we are *intellectually worried* because it seems that foundations have been gradually neglected in the same contingency in which special relativity and quantum mechanics came close to them, as was never done before, and brought them into discussion. In mathematics and physics the debate concerning foundations and incommensurability of theories seems to be ruled out by the introduction of the Hilbertan paradigm (AO, AI), set theory (AO, AI) and Bourbaki structure (AO). These formulations all dealt with classical logic (AO, AI). Rather than also considering the logic of mathematics and common knowledge, they avoided non–classical logic, which indeed could be the only alternative to the rationality and ambiguity of certain theses[1]. It also seems that a suspicion raised that the acceptance of the idea of incommensurability between theories could introduce irrationality into the system of science. Thus, with regard to Drago's assumptions, further questions rise: Is the evolution of science irrational in times of crisis? Is a gestalt evolution of science also possible? Does *normal history* exist? If so, would that kind of history also have produced a cultural *obscurantism* in historical research, especially into the foundations of science?

Perhaps, to understand the lack of attention on the foundations of science we should not look for the causes either among academics or among research teams: *La logique explore de nouvelles voies pour tenter d'analyser la créativité scientifique qui se manifeste dans l'invention et la découverte*.[2]

[1] In the first analysis one could consider Russell's studies.

[2] F. Hallyn, *Les structures rhétoriques de la science. De Kepler à Maxwell*, Seuil, Paris 2009, p. 227.